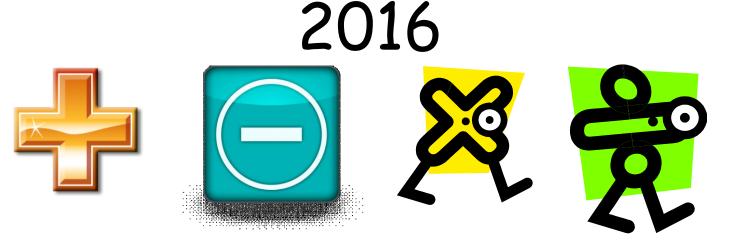




<u>Petworth C of E Primary</u> <u>School</u>

Progression in Calculation



What you need to know about calculations

Mathematics will be at the core of your child's schooling from the moment they start to the moment they leave. They will be involved in drawing, measuring, handling data and lots of other practical activities that will help your child to understand and enjoy the subject. This booklet offers guidance to the methods used to help our pupils with calculations. The methods we are advocating are in line with the National Curriculum. We hope this will be helpful to you and that you will be able to support your child in learning by heart the basic rules which will assist in mental recall eg. number bonds and multiplication tables.

The methods that we use in school may or may not be familiar to you. Children are often confused when they ask parents for help at home and they try to teach the methods that *they* themselves were taught. Knowing how the methods in this booklet work will help you to help your children.

All staff in school work from this document so that we can ensure the consistency of our approach and can make sure that the children move onto the next step when they are ready.

The four operations that are covered by this booklet are addition, subtraction, multiplication and division. Whichever operation is being taught the child needs to experience all of these steps to completely embed their understanding.

- 1) Using objects
- 2) Using pictures
- 3) Using a numberline
- 4) Using an expanded method
- 5) Using a compact written method

Mental methods first

Children should always be encouraged to consider if a mental calculation would be appropriate before using written methods. -These are covered in the first part of each section.

Why do children need to do written calculations?

- To represent work that has been done practically.
- To support, record and explain mental calculation
- To keep track of steps in a longer task
- To work out calculations that are too difficult to do mentally

Children should be taught when it is appropriate to do an approximate or estimate first and should check with the inverse operation at the end.

By upper Key Stage 2, children should be confident in choosing and using a strategy that they know will get them to the correct answer as efficiently as possible.

What can parents do to help?

- Count with their child
- Play number games
- Involve children when taking measurements or weighing items
- Take note of numbers in real life e.g. telephone numbers, bus numbers, lottery numbers etc.
- Give children opportunities to use money to shop, check change etc.
- Talking about the mathematics in football e.g. 'How many points does your favourite team need to catch the next team in the league?'
- When helping their children calculate use the method that they have been taught

Please don't...

- Teach your children that to multiply by 10 you 'just add a zero'. you 'move the digits to the left and add a zero as a place holder'
- Tell them that you can move the decimal point. You can't. You can only move the digits to the left or to the right
- Tell them that they are doing 'sums' 'sum' is a mathematical word that means 'addition', everything else is a 'calculation'

<u>Glossary</u>

2-digit - a number with 2 digits like 23, 45, 12 or 60

3-digit - a number with 3 digits like 123, 542, 903 or 561

Addition facts – knowing that 1+1 = 2 and 1+3 = 4 and 2+5 = 7. Normally we only talk about number facts with totals of 20 and under.

Array - An array is an arrangement of a set of numbers or objects in rows and columns -it is mostly used to show how you can group objects for repeated addition or subtraction.

Bridge to ten – a strategy when using numberlines. Adding a number that takes you to the next 'tens' number.

 $\ensuremath{\text{Bus Stop Method}}$ – traditional method for division with a single digit divisor

Concrete apparatus – objects to help children count – these are most often cubes (multilink) but can be anything they can hold and move.

Dienes (purple hundreds, tens and ones blocks), Numicon, Cuisenaire rods are also referred to as **concrete apparatus**.

Column chunking – method of division involving taking chunks or groups or the divisor away from the larger number

Decimal number - a number with a decimal point

Divisor – the smaller number in a division calculation. The number in each group for chunking.

Double - multiply a number by 2

Exchanging - Moving a 'ten' or a 'hundred' from its column into the next column and splitting it up into ten 'ones' (or 'ones') or ten 'tens' and putting it into a different column

Find the difference - A method for subtraction involving counting up from the smaller to the larger number

Grid method – a method for multiplying two numbers together involving partitioning

Half - a number, shape or quantity divided into 2 equal parts Halve - divide a number by 2

Integer - a number with no decimal point

 $\ensuremath{\text{Inverse}}$ – the opposite operation. Addition is the inverse of subtraction, multiplication is the inverse of division

Long Multiplication – column multiplication where only the significant figures are noted

Number bonds to ten - 2 numbers that add together to make ten, like 2 and 8, or 6 and 4.

Number bonds to 100 - 2 numbers that add together to make 100 like 20 and 80, or 45 and 65 or 12 and 88

Numberline - a line either with numbers or without (a blank numberline). Children use this tool to help them count on for addition of subtraction and also in multiplication and divison.

Numberline Chunking - method of division involving taking chunks or groups or the divisor away from the larger number

Number sentence - writing out a calculation with just the numbers in a line E.G. 2+4=6 or $35 \div 7 = 5$ or $12 \times 3 = 36$ or 32 - 5 = 27

Ones – another term for single digit numbers. The right hand column in column methods is the 'ones' column. We used to call these 'units' **Partition** – split up a larger number into the hundreds, tens and ones. E.G. 342 – 300 and 40 and 2

Place Value – knowing that in the number 342 – the '3' means '3 hundreds', the '4' means '4 tens' and the '2' means '2'.

Quarter - a number, shape or quantity divided into 4 equal parts Recombine - for addition, once you have partitioned numbers into hundreds, tens and ones then you have to add then hundreds together, then add the tens to that total, then add the ones to that total Remainder - a whole number left over after a division calculation Repeated addition - repeatedly adding groups of the same size for multiplication

Significant digit – the digit in a number with the largest value. E.G in 34 – the most significant digit is the 3, as it has a value of '30' and the '4' only has a value of '4'

Single digit – a number with only one digit. These are always less than 10. **Taking away** – a method for subtraction involving counting backwards from the larger to the smaller number

Tens number - a number in the ten times tables - 10,20,30,40 50,etc.

Resources that your children will use to help with calculation

<u>Dienes</u>



Dienes, although it has been used in schools for years is a crucial step in knowing what a 'one', a ten, a hundred and a thousand look like and how they can be added together and split up to form smaller and larger numbers.

Numicon



Numicon is an especially useful resource as it can be used for teaching all four operations as well as fractions, decimals, percentages and a range of other aspects of maths. Each piece represents an integer from 1 to 10. The children love using it as it is colourful and tactile

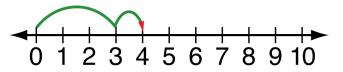
Cuisenaire Rods and a number track



Although these little rods that represent integers from 1 to 10, can be used for a range of aspects of maths, we normally use them for multiplication and division. They are also really useful for addition.

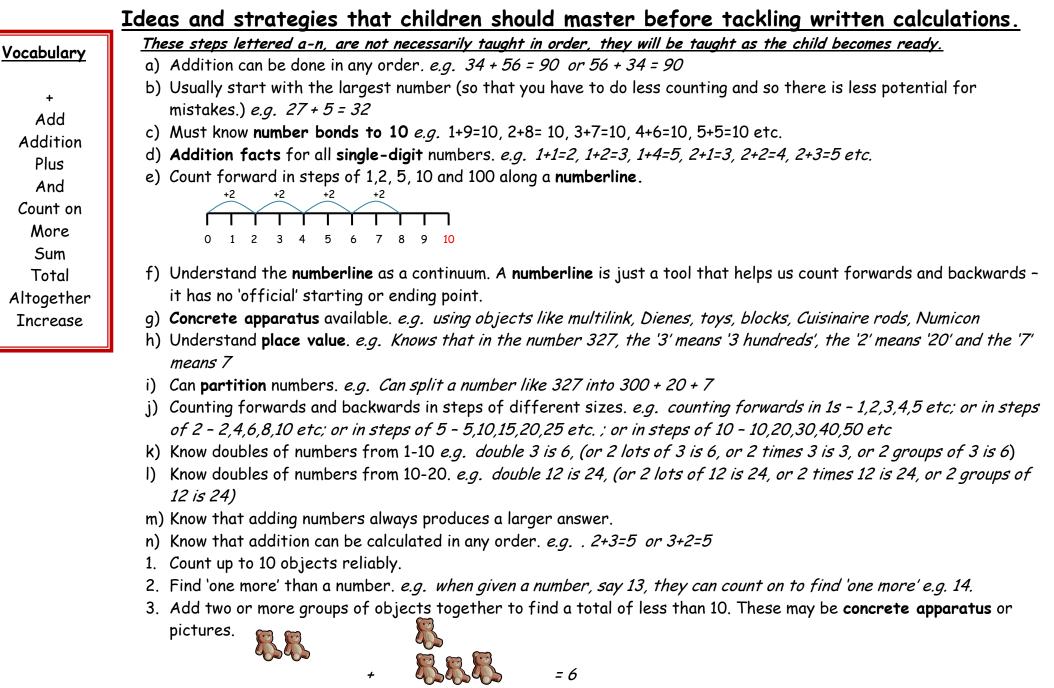
Numberlines

$$3 + 1 = 4$$



Numberlines are a mainstay of teaching calculations. We have pre numbered and blank numberlines in school that children can write on, or they can draw their own as appropriate for the calculation.

Addition



4. Use the + and = signs to record mental calculations in a number sentence. e.g. 2+6=8

Addition

Non-standard methods

- 5 Count along a **numberline** to add **single digit** numbers together to find a total of less than 10 *e.g.* 5 + 4 = 9
- 6. Add single digit numbers that bridge to 10 using a numberline. This involves partitioning the smaller number in to 2 parts, one of which will add to the larger number to make 10 e.g. 8 + 5 = 13
- 7. Add a 2-digit number and a single digit number using a numberline *e.g.* 13 + 5 = 18
- Add two 2-digit numbers bridging to 10 using a numberline. This involves partitioning the smaller number into 2 or more parts, one of which will add to the larger number to make a link to the 'next tens number' e.g. 13+15 = 28 So split 13 into 5 and 5 and 3.
- 9. Add two **2-digit** numbers adding the most **significant digit** first using a **blank numberline**. *e.g.* 42 + 35= 77
- 10. Partition and recombine e.g. 15 +13= 28 (MAY BE DONE OUT OF ORDER)
- 11. Add a 3-digit number and a 2-digit number using a numberline e.g. 243+64

Standard Written methods - Column Addition

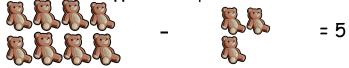
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				+2		+	3								
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5	6	7	8	9	10	11	12	13	14	15	16				
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10	11	12 +5	13	14	15	16	17 +5	18	19	20	21 +3				
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<u>12. 2-digit add 1-digit</u>	13. 3-digit add 1-digit	<u>14. 2-digit add 2digit</u>	<u>15. 3-digit add 2-digit</u>	<u>16. 3-digit add 3-digit</u>	17. Adding decimals
24	247	26	129	126	126.41
+ 5	+ 6	+ 3 8	+ 42	+ 3 5 6	+ 36.82
29	2 5 3	6 4	1 7 1	482	163.23
		X		1	XX

Subtraction

<u>Vocabulary</u>	These steps lettered a - f, are not necessarily taught in order, they will be taught as the child becomes ready.
	Subtraction can be seen in two ways: as 'taking away' or as 'finding the difference'. a) 'Taking away' is usually used when subtracting a small number from a much larger one - usually 2-digit subtract a single digit like
Less	 32 - 6. This is sometimes called 'counting back. 'At Petworth Primary School we will use this method in mental calculations only, a the 'finding the difference' method has more explicit links to more complex subtraction used further on in school. b) Must know number bonds to 10 and the reverse. e.g. 1+9=10, 2+8+10, 3+7=10 etc and 10-1=9, 10-2=8, 10-3=7 etc c) Must know number bonds to 100 (sometimes called complements to 100) e.g 20+80 = 100, 45+55=100, 100-43=57, etc d) Understand the numberline as a continuum. A numberline is just a tool that helps us count forwards and backwards - it has no 'official' starting or ending point. e) Subtraction cannot be calculated in any order. e.g. 9-4=5 is not the same as 4-9 = -5 f) Understand place value. e.g Knows that in the number 327, the '3' means '3 hundreds', the '2' means '20' and the '7' means 1. Use concrete apparatus to physically 'take away' from numbers less than 10. 2. 'Finding the difference' by counting on. By using a numberline, fingers or other apparatus or mentally count from a smaller number to a larger one. e.g. 9-4=5. Start at 4 and count on to 9. The 'difference' is the answer.

3. Use concrete apparatus or pictures to either 'take away' or 'find the difference' between 2 groups e.g. 8-3=5



- 4. Count on/count back in 1s or 10s on a numberline
- 5. Counting forwards / backwards in steps of different sizes. e.g. counting in 1s, 2s,5,s,10s etc. from any given starting point
- 6. Find 'one less' than a number. e.g. when given a number, say 13, they can count back to find 'one less' e.g. 12
- 7. Use and = signs to record mental calculations in number sentences. e.g. 23-6 = 17
- 8. Addition/Subtraction inverses (trios)

$$\begin{array}{c}
11-6 = 5 \\
11-5 = 6 \\
5+6=11 \\
\end{array}$$

Subtraction

2) There are

3 'tens' in 34.

Change one of

the tens into

ten 'ones' and

add them to

the '4 ones'.

14 - 8.

Now you have

You still have

34 in total but instead of

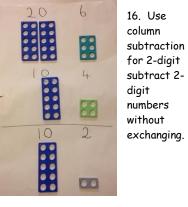
being '30+4' it is now '20 +14'

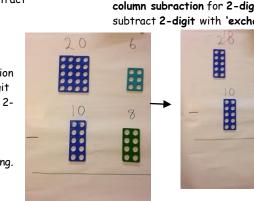
Non-standard methods

- 9 Use 'counting on' with a numberline from a single digit less than 10 to another single digit less than 10 e.g. 9-4=5
- 10 Use 'counting on' with a numberline from a single digit to a 2-digit number less than 20. e.g. 19-8=11
- 11. Subtract 10 from a 2-digit number using 'counting on', on a numberline. e.q. . 26- 10=16
- 12. Use a numberline for 2-digit numbers subtract 2-digit numbers using 'bridging to ten'. e.g. 34-15 = 19
- 13. Use a numberline for 2-digit numbers subtract 2-digit taking bigger jumps to be more efficient e.g. 76 - 29 = 47
- 14. Use numberline for 3-digit numbers subtract 2-digit numbers using efficient bigger jumps. e.g. 234 - 57 =

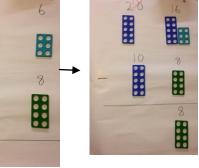
Standard written methods - Column Subtraction

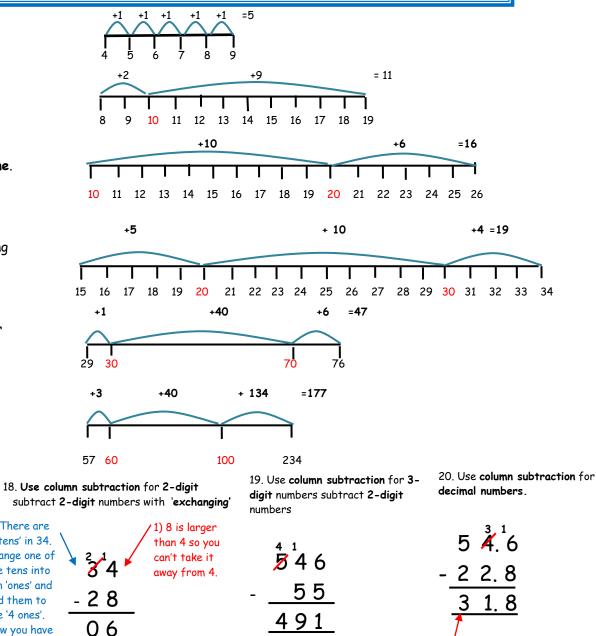
15. Use Numicon or Dienes set out in column subraction for 2-digit subtract 2-digit without 'exchanging'





17. Use Numicon or Dienes set out in column subraction for 2-digit subtract 2-digit with 'exchanging'





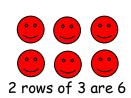
Remember to put the decimal point in your answer space first!

Multiplication

Ideas and strategies that children should master before tackling written calculations.

Vocabulary

- <u>These steps lettered a-h, are not necessarily taught in order, they will be taught as the child becomes ready.</u> a) Understand place value. e.g. Knows that in the number 327, the '3' means '3 hundreds', the '2' means '20' and the '7' means 7
- b) Recognise simple sequences of numbers. e.g. 5,10,15,20 (add five each time or count in 5s) 2,4,6,8 (add 2 each time or count in 2s)
- c) Be able to use a method for adding and subtraction (see previous sections)
- d) Know that multiplication can be calculated in any order e.g. 3x4=12 and 4x3=12
- e) Be able to show multiplication facts using arrays. You can show a number, e.g. 6, in several ways using pictures or objects



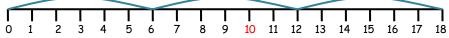
3 rows of 2 are 6



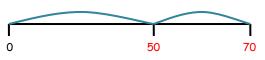
- f) That multiplication and division are **inverse** of each other. e.g. $2 \times 6 = 12$ and $12 \div 6 = 2$
- g) Can double and halve numbers from 1 to 100 e.g. Double 4 is 8, 4x2=8; half of 8 is 4, 8+2=4
- h) Multiplication is repeated addition. e.g. To find 4 x 3, you add 4 groups of 3, or you add 3 four times :3+3+3+3 =12
- 1. Put objects into groups of the same number.
- 2. Use a e.g. and Cuisenaire rods, or Numicon to multiply using repeated addition. e.g. 4 x 5 = 20



- 3. Use a numbertrack and Cuisenaire rods or Numicon alongside a numberline for repeated addition
- 4. Use a numbered numberline and record the jumps (how many groups of ..) for single digit times single digit numbers e.g. 3 × 6 1 group of 6 1 group of 6 = 3 groups of 6



- 5. Use a numberline for single digit numbers times single digit numbers
- 6. Use a numberline for 2-digit numbers times single digit numbers e.g. 14×3
- 7. Use times tables facts to make more efficient jumps on a numberline e.g. for 14 x 5, you could add 10 x 5 and 4x5 10 groups of 5 4 groups of 5

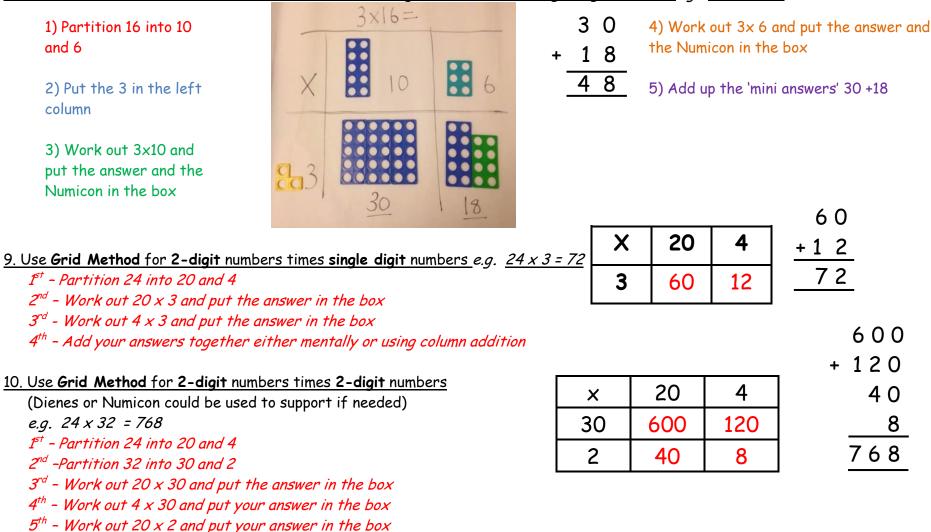


Х Lots of Groups of Times Multiply Multiplication Jumps Multiple Numberline Product Twice Three times Array Row Column Double Repeated addition

Multiplication

Non-standard methods

8. Use Dienes or Numicon set out in Grid method for 2-digit numbers times single digit numbers e.g. 3 x 16 = 48



- 6^{th} Work out 4x2 and put your answer in the box
- 7th Add your answers together using column addition

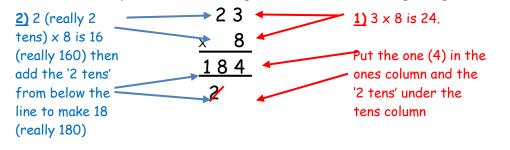
11. Use Grid Method for 3-digit numbers times 2-digit numbers

12. Use Grid Method for 3-digit numbers times 3-digit numbers

Multiplication

Standard Written Methods

13. Short Multiplication for 2-digit numbers times single digit numbers e.g. 23 x 8



14.Long Multiplication for 2-digit number times 2-digit numbers e.g.	<u>. 23 x 18</u>
4) 1x3 (really 10x3) is 3 (really 30) - Write the 3 in the	23
tens column.	× 18
5) 1×2 (really 10×20) is 2 (really 200) Write the 2 in the	184
hundreds column	230
6) Add up both of your 'mini answers'	<u>414</u>
	X

1) 8×3 is 24. Write the 4 in the ones column and the 2(really 2tens) under the tens column.

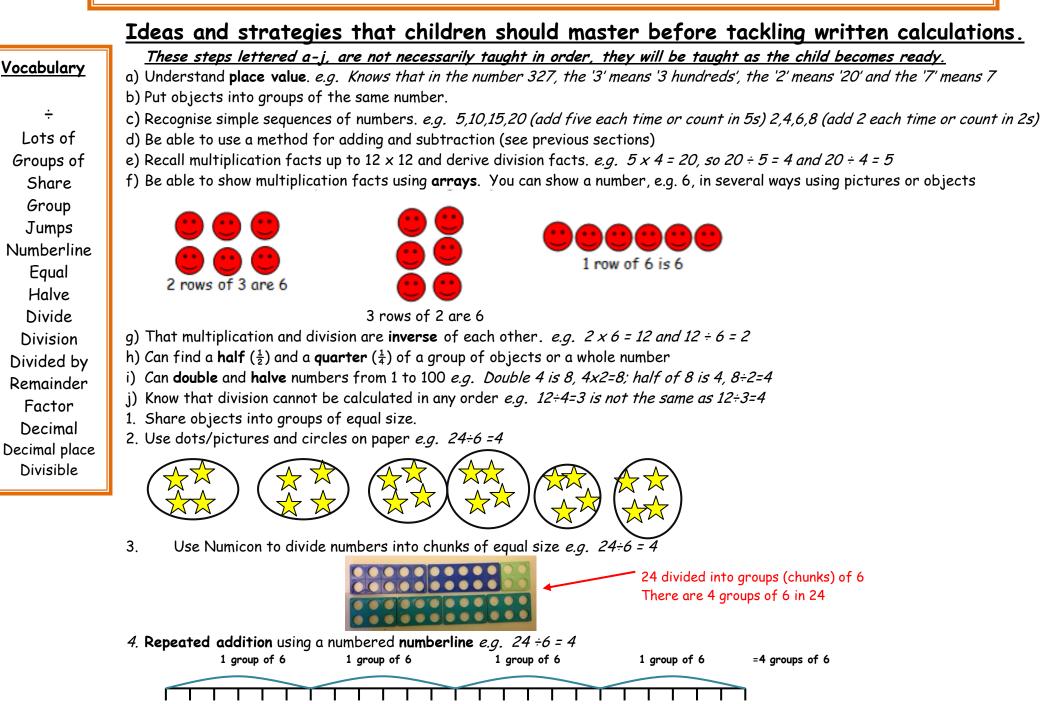
2) 8×2(really 8×20) is 16 (really 160) add the 2 tens from below the line to make 18 (really 180)

3) Place a '0' in the ones column as everything will now be multiplied by a 'tens number'.

15. Long multiplication for 3-digit numbers times 2-digit numbers. e.g. 234 x64

16. Long multiplication for 4-digit numbers times 2-digit numbers e.g. 2345x 64

17.Long multiplication for decimal numbers. e.g. 23.4 x 64.7



Non-standard methods

Chunking is a type of division with several methods.

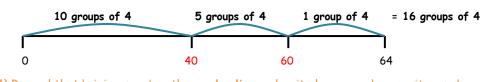
Numberline Chunking

5. Use Numberline Chunking for 2-digit numbers divided by single digit numbers e.g. $64 \div 4=16$



 Draw a numberline starting at 0 and ending with the 'target number' (64)

3) Choose the 'mini answer' from the table that is closest to the 'target number' without going over



<u>2 x4</u>	<u>5×4</u>	<u>10 x4</u>
8	20	40
<u>20 x 4</u>	<u>50 × 4</u>	<u>100 x4</u>
80	200	400

4) Record that 'mini answer' on the **numberline** and write how many '**groups** it was above the jump.

5) Which 'mini answer' can you add now to get closest to the 'target number' without going over?
6) Repeat until you end at the 'target number'. Count up how many 'groups' you have added. This is the answer.

6. Use Numberline Chunking for 3-digit numbers divided by single digit numbers with remainders (using more efficient jumps) e.g. 177÷5

30 groups of 5 5 groups of 5 remainder 2	• • •			
	<u>2 x5</u>	<u>5×5</u>	<u>10 x5</u>	
I I I 0 150 175 177	10	25	50	
7. Use Numberline Chunking for 3-digit numbers divided by 2-digit numbers with remainders.	<u>20 x 5</u>	<u>50 x 5</u>	<u>100 x5</u>	

Column Chunking (current Year 4,5 and 6)

8) Use Column Chunking for 2-digit numbers divided by single digit numbers e.g. $64 \div 4 = 16$

- 1) Use a table to write down the times tables facts for the divisor (4)
- 2) Write the largest number and then choose which is the biggest group of 4 you can subtract (10groups of 4 = 40). Subtract this from the 64.
- Write the answer and then choose 'mini answer' you can subtract next (5 groups of 4 = 20)
- 6)Repeat until you are left with 0, then count up how many 'groups' you subtracted (16 groups of 4)

4 0 (10 groups of 4)

64

24

- <u>20(5 groups of 4)</u>
- <u>4 (1 group of 4)</u>

<u>2 x4</u>	<u>5×4</u>	<u>10 x4</u>
8	20	40
<u>20 x 4</u>	<u>50 × 4</u>	<u>100 x4</u>
80	200	400

250

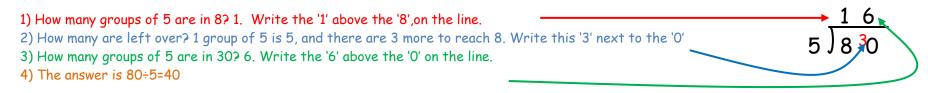
500

100

9. Use **Column Chunking** for **3-digit** numbers divided by **single digit** numbers with **remainders** (using more efficient chunks) 10. Use **Column Chunking** for **3-digit** numbers divided by **2-digit** numbers with **remainders**.

Standard Written Methods

11. Use the **Bus Stop Method** to divide a **2-digit** number by a **single digit** number *e.g.* 80÷5 = without remainders



12. Use the Bus Stop Method to divide a 2-digit number by a single digit number with remainders e.g. . 83 ÷5 = 16r3

$$\frac{1 \ 6 \ r^3}{5 \ 8 \ 3}$$

13 Use the Bus Stop Method to divide a 3-digit number by a single digit number with remainders e.g. , 483 -	<i>÷5 = 96r3</i>
14. Use the Bus Stop Method to divide a 3-digit number by a single digit number with a decimal answer e.g.	<u>483 ÷5 = 16.6</u>
 Complete the steps until you reach the point where there would be a remainder THEN Put a decimal point and two '0' after the big number Put a decimal point after the last number on the line. How many groups of 5 are in 30? '6'. Write the '6' above the line. 	$\begin{array}{r} 096.6 \\ 5 4^{4}8^{3}3.^{3}00 \end{array}$

15. Use the Bus Stop Method to divide a 4-digit number by a single digit with a decimal answer eg. 5483÷5

16. Use the Bus Stop Method to divide a decimal number by a single digit number with a decimal answer e.g. 83.7 ÷5 = 16.74

17. Use Long 'Bus Stop' Division to divide a 3-digit number by a 2-digit number with a decimal answer e.g. 462÷13 = 35.53

1) Set out the numbers for the calculation (divisor on the left) and put in a decimal point and two 'O's $13 4 6 2.00$	2) How many groups of 13 are in 4? None. Write a '0' above the 4. 3) How many Groups of 13 are in 46? 3. Write a '3' above the '6' 03 $13 \sqrt{462.00}$
4) What is 3×13 ? 39. Write this '3'9 underneath the '46' and subtract it. Write the answer '7' underneath the '9' $ \begin{array}{r} 0 & 3 \\ 13 & 13 \\ \hline 4 & 6 & 2. & 0 & 0 \\ (3 \times 13 = 39) & - & \frac{3 & 9}{7} \\ \end{array} $	5) Bring down the '2' and write it next to the '7' $ \begin{array}{r} & 0 & 3 \\ & 13 & 4 & 6 & 2 & 0 & 0 \\ & (3 \times 13 = 39) & - & 3 & 9 & 7 & 2 \\ \end{array} $
6) How many groups of 13 are there in 72? 5. Write the '5' above '2' on the answer line $ \begin{array}{r} 0 3 5 \\ 13 4 6 2.00 \\ (3 \times 13=39) - 39 \\ \hline 7 2 \end{array} $	7) What is 5 x 13? 65. Write '65' below the '72' and subtract it. Write the answer '7' underneath the '5'. $\begin{array}{c} 0 & 3 & 5 \\ 13 & 4 & 6 & 2 & 0 & 0 \\ (3 & x13=39) & - & 3 & 9 \\ \hline 7 & 2 \\ (5x13=65) & - & 6 & 5 \\ \hline 7 \end{array}$
8) Put the decimal point into the answer line. 9) Bring down the '0' and write it next to the '7' $ \begin{array}{r} 0 & 3 & 5.\\ 13 & 4 & 6 & 2. & 0 & 0\\ (3 \times 13 = 39) & - & 3 & 9 \\ \hline 7 & 2 & & & \\ \end{array} $	10) How many groups of 13 are in 70? 5. Write the '5' on the answer line above the '0' $ \begin{array}{r} 035.5\\ 13)462.00\\ (3 \times 13=39) - \underline{39} \\ 72 \end{array} $
$(5 \times 13 = 65) - 6 \frac{5}{70}$	$(5 \times 13 = 65) - \frac{65}{70} + \frac{70}{70}$

11) What is 5 x13? 65. Write the 65 below the 70 and subtract it. Write the answer	12) Bring down the next '0' and write it next to the '5'
5 underneath the '5'.	
$\frac{035.5}{12}$	035.5
$ \begin{array}{c} 13 4 6 2.00 \\ (3 \times 13 = 39) - 39 \\ \overline{72} \end{array} $	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
$(5\times13 = 65) - \frac{65}{70}$	$(5\times13 = 65) - \frac{65}{70}$
(5×13=65) - <u>65</u> 5	(5×13=65) <u>- 6 5</u> 5 0
13) How many groups of 13 are in 50? 3. Write the '3' above the '0' on the answer line.	14) What is 3×13 ? 39. Write '39' below the '50' and subtract it. Write the answer '11' underneath the '5'.
$\begin{array}{r} 035.53\\ 13 462.00\end{array}$	0 3 5. 5 3
$(3 \times 13 = 39) - 39 \downarrow$	13 4 6 2.00
72	$(3 \times 13=39)$ $-\frac{39}{72}$
$(5 \times 13 = 65) - \frac{65}{7} \checkmark$	(5x13 = 65) - 65
/ U (5v13-65)	$(5 \times 13 = 65)$ $- \frac{65}{70}$
$(5\times13=65)$ $- 65 = 50$	(5×13=65) - <u>65</u>
	$(3\times13=39) \qquad \frac{50}{-39} \\ \frac{-39}{-11}$
15) Now there are two decimal places in the answer, you can stop working	16)unless you are going to find 3 decimal places and then round to 2 decimal places

18. Use Long 'Bus Stop' Division to divide a 3-digit decimal number by a 2-digit number with a decimal answer e.g. 462.7÷13 = 35.59 19. Use Long 'Bus Stop' Division to divide a 4-digit decimal number with a by a 2-digit number with a decimal answer e.g. 2462.7÷13 = 189.44